Exercises for Numerical Methods I

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Sheet 1

due October 14, 2015

1. Compute the matrix norms $\|A\|_{\infty,1}$, $\|A\|_2$ and $\|A\|_F$ of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

2. Let $A \in \mathbb{R}^{n \times n}$ and $x \in \mathbb{R}^n$. Show that $\|Ax\|_2 \leq \|A\|_2 \|x\|_2$, where $\|A\|_2$ is the spectral norm.

3. Compute the condition number of the matrix

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

4. Consider the linear system $Ax = b$ and its perturbation $A(x + \Delta x) = b + \Delta b$ with

$$A = \begin{bmatrix} 1 & 4 \\ 1 & 4.001 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad \Delta b = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}.$$

   Check if the system is well- or ill-posed and compute the relative error.

5. (a) Let $\{a_\varepsilon : \varepsilon \neq 0\}, \{b_\varepsilon : \varepsilon \neq 0\}$ and $\{c_\varepsilon : \varepsilon \neq 0\}$ be parametrized families of numbers satisfying $a_\varepsilon, b_\varepsilon, c_\varepsilon \neq 0$. Show that if $a_\varepsilon = \mathcal{O}(b_\varepsilon)$ and $c_\varepsilon = \mathcal{O}(a_\varepsilon)$ then $c_\varepsilon = \mathcal{O}(b_\varepsilon)$.

   (b) Show that:

      i. $\log(1 + x) = x + \mathcal{O}(x^2)$.

      ii. $\arctan(x) - x = o(x)$.

6. Consider the linear system

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= 6 \\ x_2 + 2x_3 &= 3 \\ 5x_1 + 3x_2 + x_3 &= 4 \end{aligned}$$

   Solve the above system using the Gauss -Elimination algorithm.