Exercises for Numerical Methods I

Dr. C. Gerhards   Faculty of Mathematics, University of Vienna   October 15, 2015

Sheet 2

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1. Compute the LR-decomposition of the matrix

$$A = \begin{bmatrix} 6 & 2 & 1 \\ 2 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}.$$

2. Consider the linear system

$$\begin{bmatrix} 0 & 4 & -1 \\ 3 & -1 & -1 \\ 1 & -5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \\ 11 \end{bmatrix}.$$

Solve the above system using Gauss-elimination with partial pivoting.

3. Compute the Cholesky-decomposition of the matrix

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

4. Let $v = [2, -2, 1]^T$. Compute the Householder transformation $H \in \mathbb{R}^{3\times 3}$ and verify that

   a) $H^2 v = v$

   b) $\|Hv\|_2 = \|v\|_2$

   c) $(Hw)^T Hv = 0, \quad \text{where} \quad w = [0, 1, 2]^T$.

5. Consider the linear system $Ax = b$ with

$$A = \begin{bmatrix} a_{11} & 0 & \cdots & 0 & a_{18} \\ 0 & a_{22} & \ddots & \vdots & a_{28} \\ \vdots & \ddots & a_{33} & 0 & a_{38} \\ 0 & \cdots & 0 & \ddots & \vdots \\ a_{81} & a_{82} & a_{83} & \cdots & a_{88} \end{bmatrix} \in \mathbb{R}^{8\times 8}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_8 \end{bmatrix} \in \mathbb{R}^8.$$

Write a MATLAB-Function, that for given matrix A (of the above form) and vector b solves the linear system using the substitution method. The substitution method works as follows: Solve the first seven equations for one of the variables (you choose which one) and then plug this back into the last (eigth) equation, solving this quation for the chosen variable. The remaining variables are then computed from the previous equations.

6. We define the **golden ratio** ϕ as the positive solution of the quadratic equation $\phi^2 - \phi - 1 = 0$. Using one of the following iteration methods

$$a) \quad \phi_{n+1} = 1 + \frac{1}{\phi_n}$$

or

$$b) \quad \phi_{n+1} = \frac{\phi_n^2 + 1}{2\phi_n - 1}$$

we can approximate the golden ratio as $\phi = \lim_{n \to \infty} \phi_n$. We set as initial value $\phi_0 = 1$. The iterations are terminated if the stopping criterion

$$\|\phi^* - \phi_n\|_2 \leq 10^{-4}, \quad \text{where} \quad \phi^* \quad \text{is the exact solution,}$$

is satisfied. Create a MATLAB-Program that implements the two iteration schemes for approximating ϕ. Which iteration method converges faster?