Exercise Sheet 4

1. A company produces a product that does not break under a given force with probability 0.8. We choose 9 random products and we apply the given force to each of them. What is the probability of:
   (a) at least 7 unbroken products?
   (b) at most 2 unbroken products?
   (c) less than 8 unbroken products?
   (d) less than 6 and at least 4 unbroken products?

2. Consider the probability mass function $p_S(k)$ of the binomial distribution with parameters p and n. Find a coefficient $c = c(p, n, k)$ such that the iteration scheme

$$p_S(k) = c\, p_S(k-1), \quad k = 1, 2, ..., n$$

   holds. Find the value of k that is more probable to appear. Justify the result for p, n as in exercise 1.

3. A laboratory test is repeated until the first success. The tests are independent with probability of success 3/4. The first test costs 40 € and due to some necessary modifications for every next test an additional amount of 5 € is needed. Compute:
   (a) the probability of at most 4 tests until the first success.
   (b) the expected total cost until the first success.

4. A fisherman catches a fish according to a Poisson process with rate $\lambda = 7$ per hour. Compute the probability:

   (a) that he catches at most 3 fish.

   (b) that in a random 20min interval he catches at least 2 fish.

5. Consider the hypergeometric distribution with probability mass function

$$p_H(k) = \frac{\binom{m}{k}\binom{n}{l-k}}{\binom{m+n}{l}}, \quad \text{for} \quad \max\{0, l-n\} \leq k \leq \min\{m, l\}$$

   describing the probability of k successes in $m+n$ trials, without replacement. We set $N = m + n$. If $N, m, n \to \infty$ such that

$$\lim_{N\to\infty} \frac{m}{N} = p \in (0, 1),$$

   show that

$$\lim_{N\to\infty} p_H(k) = p_S(k),$$

   where p_S is the PMF of the binomial distribution with parameters p and l.