Exercise Sheet 1 (October 16th, 2015)

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Exercise 1. Find the general solution of the differential equation

$$\dot{y} = y^2 \, .$$

In addition, find particular solutions for the initial conditions $y(1) = 1, y(1) = -1$ and $y(1) = 0$, respectively.

Exercise 2. Find the general solution of the differential equation

$$(t^2 + 1)\dot{y} + ty = \frac{1}{2} \, .$$

Exercise 3. Find the general solution of the differential equation

$$(3t - y)\dot{y} + t = 3y \, .$$

Note that this equation is of homogeneous type.

Exercise 4. Implement in MATLAB the explicit Euler method, the midpoint method and Heun's method for the solution of a system of ODEs of the form

$$\dot{y}(t) = f(t, y(t)) \, , \quad y(t_0) = y_0 \, , \tag{1}$$

up to some time $T > t_0$.
Test your code on the system of ODEs

$$\dot{y}_1 = -y_2 \, , \dot{y}_2 = y_1 \, , \qquad y_1(0) = 1 \, , \quad y_2(0) = 0 \, .$$

with final time $T = 2\pi$, and compute the approximation error err_j (the norm of the difference between the solution of the ODE and its approximation) for step sizes $2\pi/2^j, j = 1, ..., 10$ (the actual solution is: $y_1(t) = cos(t), y_2(t) = sin(t)$). A numerical approximation of the order of the methods (the numerical order) can be found by observing the ratios

$$-\frac{\ln(\mathrm{err}_{j+1}/\mathrm{err}_j)}{\ln 2} \quad \text{for large } j.$$

Explain why this is reasonable and determine the numerical order.