# Exercise Sheet 3 (November 13th, 2015)

Otmar Scherzer

---

**Exercise 9.** The *implicit Euler method* for the solution of a linear ODE of the form

$$\dot{y} = Ay, \qquad y(0) = y_0;$$

is defined by the iteration

$$y_{k+1} = y_k + hAy_{k+1}.$$

here we consider $A : \mathbb{R} \to \mathbb{R}^{d\times d}$ which is given and $y : \mathbb{R} \to \mathbb{R}^d$. Implement this method and use it for the numerical solution of the mass-spring system of the stiff setting in Example 8 of Exercise Sheet 2.

**Exercise 10.** Implement the trapezoidal rule (second order Adams-Moulton method) for linear ODEs and use it for the numerical solution of the mass-spring system in the stiff setting (the same problem as above).

**Exercise 11.** Consider the following ODE of boundary value problem:

$$-\varepsilon u'' + u' = 0 \text{ in } (0,1)\,, \qquad u(0) = 0, u(1) = 1\ . \tag{1}$$

Find out the general solution. Write the numerical discretization form of (1) by taking into account the boundary conditions and choosing the uniform step size $h$ and using various finite difference approximate the derivative $u'$ (namely, forward, backward and central differences), and the central difference operator for the second derivative, which turns to a linear system

$$AU = f, \tag{2}$$

here $A \in \mathbb{R}^{(n-1)\times(n-1)}$ is a matrix, $U = (u_1, u_2, \cdots, u_{n-1})^T$ is a vector which is the unknown, representing the value of $u(x_k)$, $k = 1, 2, \cdots, n-1$. That is, you have to write out the formulations of $A$ and $f$ corresponding to different finite difference methods.

**Exercise 12.** Implement the numerical methods for solving Example 11 with $\varepsilon(x) \equiv 1/2^8$ and the boundary condition $u(0) = 0, u(1) = 1$ , and the step size is fixed on $h = 1/2^4$ uniformly. Compare the solutions by choosing the three different finite difference approximations of the first order derivative $u'$. Explain the results.