Prof. Otmar Scherzer
Dr. Peter Elbau

Numerical Analysis 2
Faculty of Mathematics

University of Vienna
January 15, 2013

Exercise Sheet 9

1. Let $T \subset \mathbb{R}^2$ be a triangle and let y_1, y_2, and y_3 denote the midpoints of its edges. Show that
$$\int_T P(x)\,\mathrm{d}x = \frac{|T|}{3}\sum_{k=1}^{3} P(y_k)$$
for every polynom P with $\deg(P) \leq 2$, where $|T|$ denotes the area of the triangle T.

2. Let Ω be a bounded, polygonal domain in $\mathbb{R}^2$ and let $\gamma > 0$. Show that there exist positive constants c and C such that we have for every $h > 0$, every regular triangulation Γ of Ω with maximal side length h and minimal area $\min_{T\in\Gamma}|T| \geq \alpha h^2$ of the triangles and every function $v \in C(\Omega)$ which is linear on every triangle $T \in \Gamma$ the estimate
$$c\|v\|_{L^2(\Omega)} \leq h\|(v(x_i))_{i=1}^n\|_2 \leq C\|v\|_{L^2(\Omega)},$$
where $\{x_i\}_{i=1}^n \subset \Omega$, $n \in \mathbb{N}$, denotes the vertices of the triangulation Γ.

—