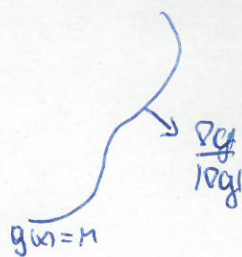


Nonlinear Diffusion

$$\frac{\partial}{\partial t} g(t, x, y) = \nabla \cdot (D(|\nabla g(t, x, y)|^2) \nabla g(t, x, y))$$

E.g. $D(|\nabla g|^2) = e^{-\frac{|\nabla g|^2}{2\sigma^2}}$, $D(|\nabla g|^2) = \frac{1}{1+\sigma|\nabla g|^2}$



$$\nabla \cdot (D(|\nabla g|^2) \nabla g) = D(|\nabla g|^2) \Delta g + (\nabla D(|\nabla g|^2)) \cdot \nabla g$$

$$= D(|\nabla g|^2) \Delta g + 2D'(|\nabla g|^2) (\nabla g)^T \text{Hess}_g \nabla g$$

$$= D(|\nabla g|^2) \Delta g + 2|\nabla g|^2 D'(|\nabla g|^2) \underbrace{\left(\frac{\nabla g}{|\nabla g|} \right)^T}_{\text{normal vectors}} \text{Hess}_g \underbrace{\frac{\nabla g}{|\nabla g|}}_{\Delta_{\text{normal}}}$$

normal vectors in x at level set boundary $\{x \in \mathbb{R}^2 \mid g(x) = h\}$

$$= \underbrace{D(|\nabla g|^2)}_{\Delta = \Delta_{\text{tan}} + \Delta_{\text{normal}}} \Delta_{\text{tan}} g + \underbrace{(4|\nabla g|^2 D'(|\nabla g|^2) + D(|\nabla g|^2))}_{\text{dominates}} \Delta_{\text{normal}} g$$

if $|\nabla g| \gg 1$, ~~the~~ (i.e. ~~normal~~ tangential diffusion) dominates

if $|\nabla g| \ll 1$, ~~the~~ $\otimes \otimes \approx \otimes$ (approx. isotropic diffusion)