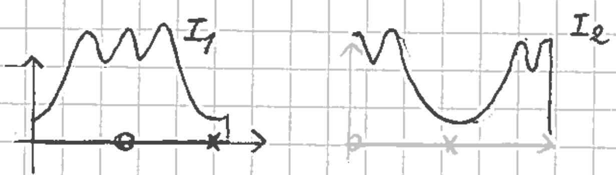
Integral Invariants
and
Shape Matching (continuation)

§5. Shape Matching, Shape Dist.
§6. Multiscale Invariants
§7. Implementation
and Exp Results

§5. Shape Matching and Shape Distance

After having explained how curves can be described by integral invariants, which in this § will be assumed to be simply functions $I(s)$ of a parameter s, we address the problem of comparing two shapes by means of such an integral invariant function. As such we should assign a measure of distance between 2 shapes ~~as~~ based on their 2 respective functions $I_1 = I_1(s)$ and $I_2 = I_2(s)$.

We first remark that this distance should be zero if the 2 functions I_1 and I_2 are the same up to a translation of the parameter s:



represent the same shape
(corresponding points are marked)

We thus search for a reparametrisation correspondence which is described by the disparity function d

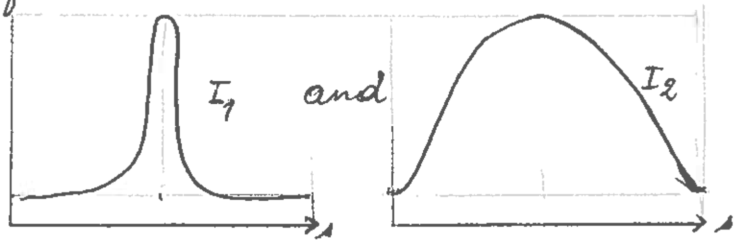
$$\underbrace{s - d(s)}_{\text{Parametrisation for first curve}} \longleftrightarrow \underbrace{s + d(s)}_{\text{Parametrisation for second curve.}}$$

The „matching" or „correspondence" between the point $s-d(s)$ left and $s+d(s)$ right should be considered as good if the integral invariants at corresponding points are approximately equal, which means that

$$\int \| I_1(s-d(s)) - I_2(s-d(s)) \|^2 \, ds \qquad (13-1)$$

is small.

Now we also want to assign a „cost" to the bending or stretching of the correspondence.
Indeed, if we would not do so, even the two shapes represented by



would become zero in the sense that there does exist a correspondence between the parameter domains whose disparity function ~~yields~~ makes (13-1) zero.
The remedy is to consider the functional

$$\mathcal{E}(I_1, I_2, d) = \int \| I_1(s-d(s)) - I_2(s+d(s)) \|^2 \, ds + \alpha \int \| d'(s) \|^2 \, ds$$

for a certain $\alpha > 0$.
The „coupling constante" α determines the „cost for reparametrisation".

Finally a <u>shape distance</u> is defined as

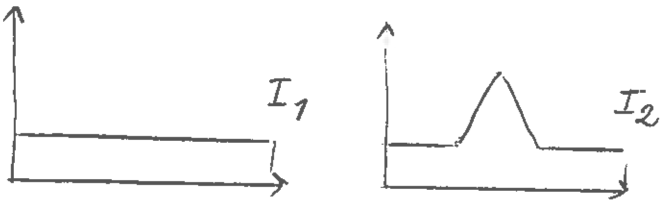
$$\mathcal{D}(I_1, I_2) = \min_d \mathcal{E}(I_1, I_2) \qquad (13-2)$$

(In the article it is more or less assumed that this minimum ~~dista~~ exists and is unique, but this need not be the case.)

The function d^* where this minimum is achieved determines an optimal correspondence or <u>shape matching</u>.

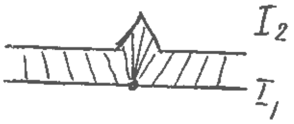
(This means, $s - d(s)$ on curve 1 should be similar to $s + d(s)$ on curve 2.)

Let us now consider two shapes with the following integral invariants:



For various values of α the optimal correspondence can be found and represented as follows:

α large



α small (reparametrising is „cheap")

We mention finally that the shape distance $\mathcal{D}$, defined in (13-2), satisfies

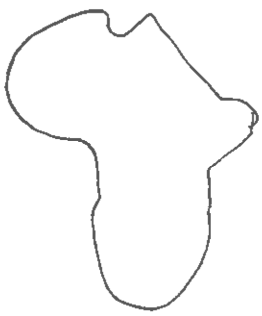
$$\mathcal{D}(I_1, I_2) = \mathcal{D}(I_2, I_1)$$

§6. Multiscale Invariants

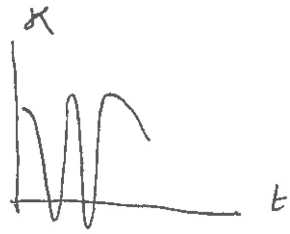
In [4] the following has been described: for a planar curve $\gamma = (x(t), y(t))$, we want to represent the values of t which correspond to an inflection point on the curve, taking into account only information of the curve on a certain level of detail σ.

Thus for a certain σ one convolves the co-ordinate functions x, y with Gaussian kernels of distribution σ, resulting in a smoother version of the curve, and then traces the inflection points of this curve. (An inflection point of the curve is a point where its curvature vanishes).

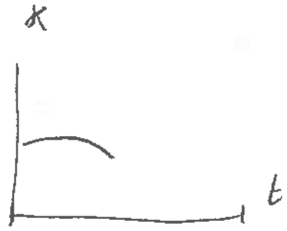
As an example one takes the coast of Africa.



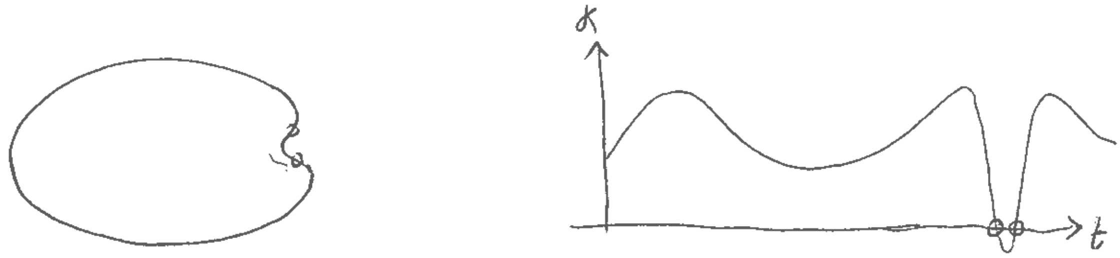
σ = small



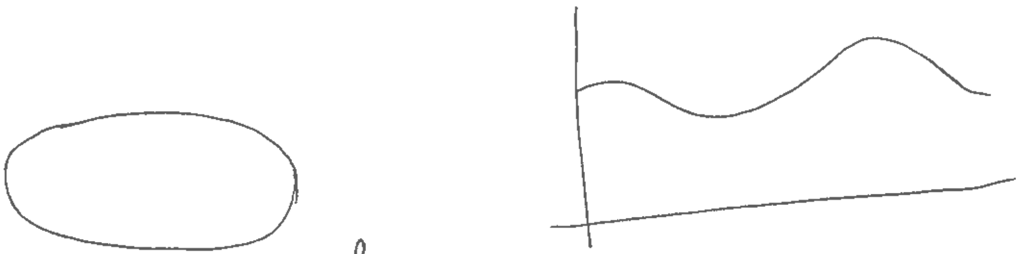
σ = large



Typically one sees that, upon taking σ larger, the number of inflection points decreases (they „cancel each other"):



σ small



σ large.

Then, one represents the figure by the map of all inflection points at all levels of detail σ:

and calls this „<u>scale space image</u>".

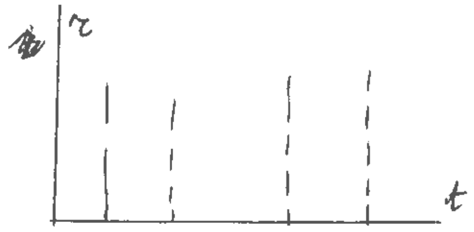
In Such a picture we typically see how, when σ increases, 2 inflection points approach and then cancel out.

Particularly the parameter value of the inflection points depends on the scale parameter σ.

In the article under discussion of Manay et al this idea is borrowed and adapted to the <u>local area integral invariant</u> I_r where r plays the role of level of detail.

Particularly one can draw e.g. the critical points of I_r (as a function of t) and this for different values of t.

Here it is claimed that these values are not dependent on r, i.e., the picture would be static:



§7. A few remarks about implementation and experimental results.

We now discuss the implementation of the concepts of §5.

Particularly: for two given functions I_1, I_2,

both of them known in a discrete # of points,

find such an optimal correspondence.

This can be translated into a graph search problem.

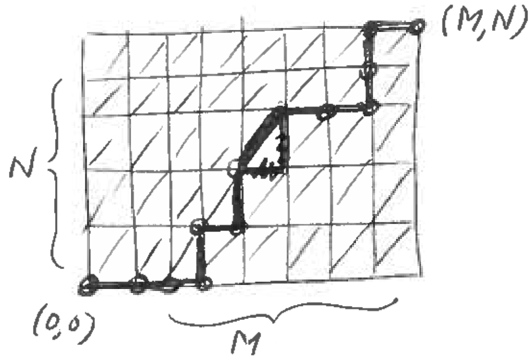
Let I_1 be known in points $0, \Delta s_1 \ldots \; M\Delta s_1 = 1$

I_2 $\quad 0, \Delta s_2 \ldots \; N\Delta s_2 = 2.$

Represent I_1- parameter horizontally

I_2- vertically

A possible correspondence between these parameter domains looks as:



thus a path from (0,0) to (M,N). (Here we assume end- and begin-points of the 2 curves to match.)

To obtain a discrete version of a (1-1) correspondence only steps to the right → or diagonal ↗ or up ↑ are allowed.

Such a discrete path is thus a sequence of vertices

$$p = (v_0, v_1, v_2 \cdots v_L) \qquad \left.\begin{array}{l} v_0 = (0,0) \\ v_L = (M,N) \end{array}\right\}$$

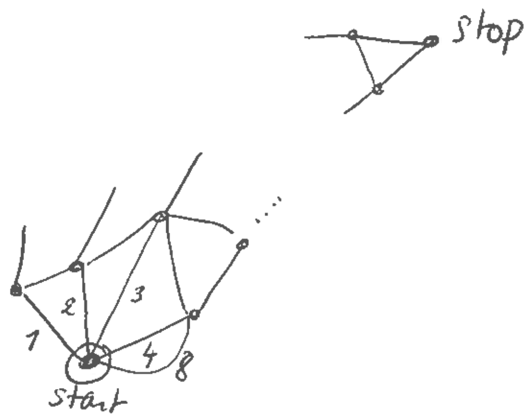
The discrete version of the functional we minimise can be written as

$$w(p) = \sum_{t=0}^{L-1} w(v_t, v_{t-1})$$

where $w(v_{ij}, v_{kl})$ is a certain ~~we~~ weight assigned to the ~~...~~ edge.

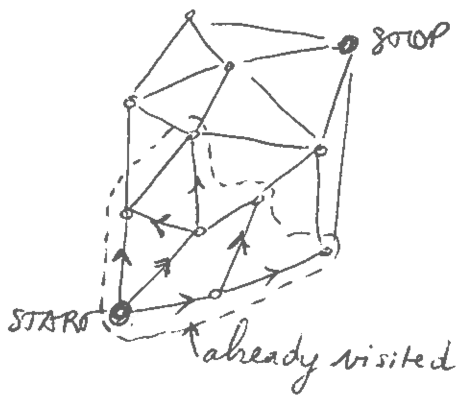
Then this can be solved (for GLOBAL minimisers) using Dijkstra algorithm

Roughly speaking this algorithm starts from the initial node with calculating.



- First only the "start" node is ~~con~~ marked.
  Set it as "current", i.e., "on the boundary of our calculations".
- Next we consider all neighbours of the current node and write down the least distance of them to the start.
- Then we consider the neighbours of the neighbours etc.

At every stage during runtime, we ~~have~~ can graphically represent our calculations as:



where for each point in the calculated domain the minimal distance to the start (within this domain) is known, as well as how to realise it.
If the entire graph is finished the solution is known.

Experimental results.

* Fig 10 OCCLUDED FINGER
* Shape matching via area integral invariant is much more robust then with curvature! Fig 11

Additional References.

[2] F. Fidler, M. Grasmair, O. Scherzer,
Identifiability and Reconstruction of Shapes from Integral Invariants.

[3] H. Pottmann, J. Wallner, Q. Huang, Y-L Yang
Integral Invariants for Robust Geometry Processing (2009).

[4] F. Mokhtarian and A. Mackworth.
Scale-Based Description and Recognition of Planar Curves and Two-dimensional Shapes.
IEEE, PAMI-8, Jan'86, pp 34-43.