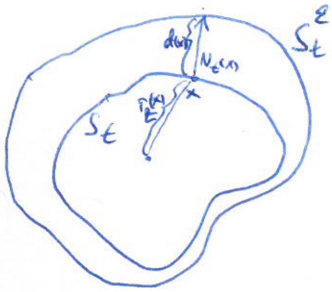
Surface Morphing   Lect 6

Functional

$$E(S_t) = \int_{S_t^{int}} \chi_{S_e^{int}}(y)\, dy$$

Compute variational derivative: Disturbed surface S_t^ε is given by

$x \in S_t \Rightarrow x + \varepsilon d(x) N_t(x) \in S_t^\varepsilon$, N_t is surface normal of S_t



$$\frac{d}{d\varepsilon} E(S_t^\varepsilon)\Big|_{\varepsilon=0} = \frac{d}{d\varepsilon}\left(\cancel{\int_{S_t^{int}} \chi_{S_e^{int}}(y)\, dy} + \int_{S_t^{\varepsilon,int} - S_t^{int}} \chi_{S_e^{int}}(y)\, dy \right)\Bigg|_{\varepsilon=0}$$

$$= \frac{d}{d\varepsilon}\left(\int_0^{2\pi}\int_0^{\pi}\int_{r_t(\theta,\varphi)}^{r_t(\theta,\varphi)+\varepsilon d(\theta,\varphi)} \chi_{S_e^{int}}(\theta,\varphi,r)\, r^2 \sin\theta\, dr\, d\theta\, d\varphi \right)\Bigg|_{\varepsilon=0}$$

assume that $S_t^{\varepsilon,int} - S_t^{int}$ is starshaped

$$= \int_0^{2\pi}\int_0^{\pi} \chi_{S_e^{int}}(\theta,\varphi,r_t(\theta,\varphi))\, \underbrace{d(\theta,\varphi)}_{=(N_t(\theta,\varphi)\, d(\theta,\varphi))\cdot N_t(\theta,\varphi)}\, r_t(\theta,\varphi)^2 \sin\theta\, d\theta\, d\varphi$$

$$= \int_{S_t} \chi_{S_e^{int}}(x)\, N_t(x) \cdot (N_t(x)\, d(x))\, dS(x)$$

$$\Rightarrow \nabla E(S_t^\varepsilon) = \chi_{S_e^{int}}(x)\, N_t(x)$$