Visualization and Imaging

Exercise Sheet 2

Exercise 1:

Remember our setting for volume rendering via ray casting. Derive integral representations of the intesity I in the image plane for the following different scenarios:

(a) two light sources that contribute via diffuse and specular reflection, single scattering, no shadows, no ambient ligth

(b) two light sources that contribute via diffuse and specular reflection, single scattering, shadows for both light sources, no ambient ligth

(c) two light sources that contribute via diffuse and specular reflection, multiple scattering, no shadows, no ambient ligth

Exercise 2:

Assume we are given a regular point grid $\{x_{i,j}\}_{i,j=1,\dots,N} \subset [0,1]^2$ with costant grid spacing $r > 0$ (i.e., $|x_{i,j} - x_{i+1,j}| = |x_{i,j} - x_{i-1,j}| = |x_{i,j} - x_{i,j+1}| = |x_{i,j} - x_{i,j-1}| = r$). Furthermore, we know a radial basis function $f : \mathbb{R}^2 \to \mathbb{R}$ with support in $B_{2r}(0)$, i.e., $f(x) = 0$ of $|x| \geq 2r$ and $f(x)$ only depends on $|x|$. Let now $\tilde{I} : \mathbb{R}^2 \to \mathbb{R}$ be of the form

$$\tilde{I}(x) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{i,j} f(x - x_{i,j}), \quad x \in [0,1]^2,$$

and interpolate a measured intensity $I_{i,j}$ in the grid points $x_{i,j}$, i.e.,

$$\tilde{I}(x_{i,j}) = I_{i,j}, \quad i,j = 1,\dots,N.$$

Compute a kernel $K : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ such that

$$\tilde{I}(x) = (K * I)(x) = \sum_{i=1}^{N} \sum_{j=1}^{N} K(x, x_{i,j}) I_{i,j}, \quad x \in [0,1]^2.$$

Exercise 3:

Consider the integral expression

$$I(t) = I_0 e^{-\int_0^t f(s)ds} + \int_0^t g(s) e^{-\int_s^t f(r)dr} ds$$

and let $0 = t_0 < t_1 < \ldots < t_n = t$ be a subdivision of the interval $[0,t]$. Show that we can compute $I_i = I(t_i)$ iteratively by the recursion formula

$$\begin{aligned}
T_0 &= 1 \\
T_i &= e^{-\int_{t_{i-1}}^{t_i} f(s)ds} T_{i-1} \\
I_i &= T_i I_{i-1} + \int_{t_{i-1}}^{t_i} g(s) e^{-\int_t^{t_i} f(r)dr} ds.
\end{aligned}$$